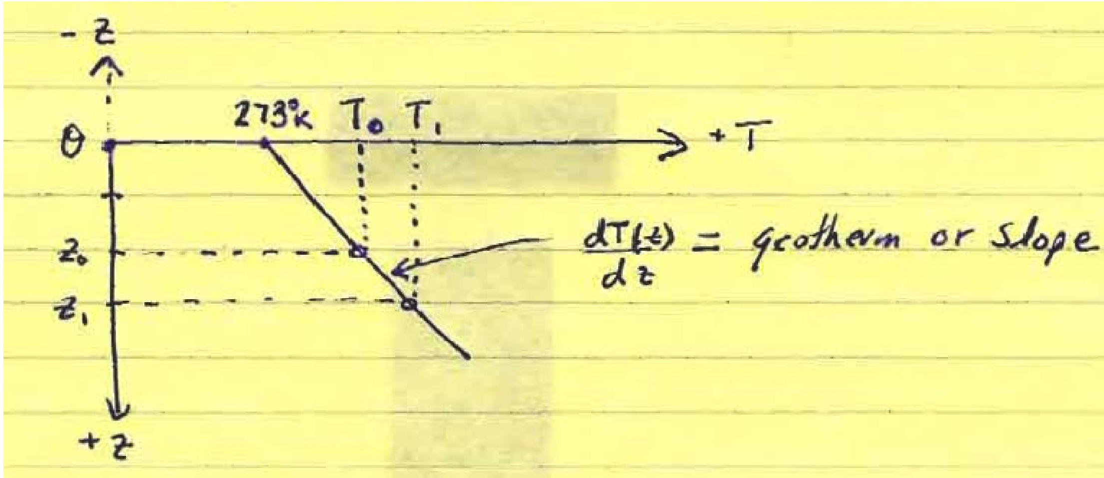
**Introduction to Geophysics: Heat flow lab** Name ______________________________




## Equations

°K =°C + 273.15°; You will have to shift units from km/m and °K/°C in this lab.

(0.1) $$q(z) = -K(z)\frac{dT(z)}{dz} \quad W/m^2 \ (Power/area)$$

(0.2) $$\frac{dT(z)}{dz} \approx \frac{T_1 - T_0}{z_1 - z_0} \approx \frac{\Delta T}{\Delta z} \qquad K^\circ / m$$

(0.3) $$T(z) = \int_0^z \frac{-q(z)}{K(z)}\, dz + 273^\circ K$$

**Let's solve the heat flow equation for the geotherm function T(z).**

$$q(z) = -K(z)\frac{dT(z)}{dz} \qquad \frac{dT(z)}{dz} = \frac{-q(z)}{K(z)} \qquad dT(z) = \frac{-q(z)}{K(z)}dz$$

$$\int_{T(z'=0)}^{T(z'=z)} dT(z') = \int_0^z \frac{-q(z)}{K(z)}dz \qquad T(z) - T(z=0) = -\int_0^z \frac{q(z)}{K(z)}dz$$

In a borehole, you measure the following temperatures: 25°C at 1 km depth and 50°C at 2 km depth and 75°C at 3 km depth.

1. (1 pt) Calculate a numeric estimate of dT/dz (first derivative or ΔT/Δz) in both MKS and CGS units.

2. ( 1 pt) Calculate the heat flow at the surface (z=0 m). Use heat flow equation (1.1) and assume the thermal conductivity K(z) is a constant function: K(z) = 2 W/m-°K.

3. (2 pt) Knowing that K(z) is always a positive value, then the sign of the geotherm (dT/dz) is the only term that can change the sign of the heat flux. That is,

   If dT/dz > 0, q is negative.

   If dT/dz < 0, q is positive.

   For this one-dimensional problem, the sign of the heat flux specifies the direction of heat flux. Note that in the graph above I have defined up as the minus z direction and down as the plus z direction. Rearranging equation 1.1 gives:

(0.4)
$$\frac{dT(z)}{dz} = -\frac{q(z)}{K(z)}$$

   Draw a graph of temperature (dependent variable) versus of the independent variable depth (z) for two geothermal gradients: (a) dT/dz < 0; (b) dT/dz > 0.

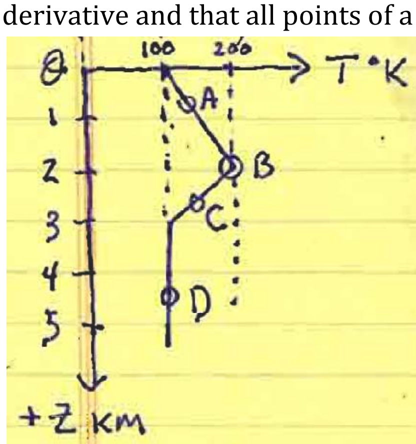
4. (2 pt) Using equation 1.1 and assuming that q(z) is a constant non-zero value, mathematically show the value the geotherm dT/dz approaches for two cases.
   (a) The perfect heat conductor case where K(z) → ∞ .
   (b) The perfect heat insulator case where K(z) → 0 .

5. (2 pt) Assume the heat flux and thermal conductivity functions are constant: q(z) = C;  K(z) = D.  Use equation 1.3 to solve for the geotherm function T(z) using integral calculus.

6. (1 pt) Assume K(z) = 2 W/m-°K and T(z=0 m) = 273°K and q(z=0 m) = -50 mW/m$^2$, calculate the temperature at 10 km depth.

7. (1 pt) Analytically calculate the first order derivative of the geotherm function T(z) in problem 5. In words, what is the physics of the first derivative.

8. (3 pt) Given the geotherm graph, numerically calculate the first derivative dT/dz and the heat flux (q) at the A-D points labeled. Specify the direction of the heat flow at each point. Hint: remember the definition of a derivative and that all points of a continuous function are not always differentiable.



9. (2 pt) (a) Assume no heat production and that the surface (z=0) temperature is fixed at 100°K. (a) Is the above geotherm a steady-state geotherm? (b) Let time → ∞, draw the geotherm and calculate heat flow assuming K(z) = 2 W/m-°K.

10. (2 pt extra credit) The one-dimensional time-dependent heat flow equation can be written as:

$$\rho C_p \frac{dT(z,t)}{dt} = -k \frac{d^2T(z,t)}{dz^2} + \rho H(z) \qquad W / m^3 \; (Power / volume)$$

Do a dimensional analysis to show that the units of the three different terms are equal. Heat production H has units of W/kg and density $\rho$ has units of kg/m$^3$ and heat capacity $C_p$ has units of J/kg-K and thermal conductivity *k* has units of W/m-K. Show that the following function solves the time dependent heat flow equation when the heat production function H(z) is zero.

$$T(z,t) = T_o e^{-at} e^{-bz} \qquad \text{where a, b, and } T_o \text{ are constants, t is time and z is depth.}$$

First calculate the necessary derivatives of *T(z,t)* , then substitute these derivatives into the equation without the heat production term. After canceling terms, you will be left with a simple algebraic equation of the following variables: $\rho, a, b, k, C_p$. If the solution is correct, the units of the algebraic equation should be dimensionally correct.